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Application of Multivariable Linear Control Design to Marine Towed Systems

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Model-based design and testing of controllers for underwater towed systems are discussed. The application of the linear quadratic Gaussian/loop transfer recovery (LQG/LTR) control design methodology to the development of depth, roll, and yaw rate controllers for a vehicle towed by a single tow cable is described. Software tools used for model generation and for testing the robustness of designed controllers are also described. The software allows controllers to be tested, for example, against model uncertainties, sensor noise, tow vessel motion disturbances, simulated unsteady vehicle hydrodynamics, actuator dynamics, and nonlinear vehicle hydrodynamics. The software shows that with the appropriate design parameters, the LQG/LTR methodology can produce robust controllers. This observation has generally been backed by sea-trial experience.

Introduction

THE U.S. Navy has been developing mine countermeasures for many years to make the waterways safe for ship traffic. One example of these systems consists of a single vehicle pulled by a tow cable; the vehicle houses sonars that are used to identify and locate mines on or moored to the ocean floor. The vehicle itself will generally have a wing to generate depressive force and a tail with flaps for control. For such a system to be viable, the following high-level design specifications must be met. First, the system must be towable at various speeds, and especially at high speeds (greater than 10 kn) for acceptable area coverage rates. Second, for operation in different water depths, the system must be operable at various tow scopes and at various depths for each of those scopes. Finally, for effective sonar operation, the vehicle must maintain a nearly constant position relative to the towing vessel and must have minimal angular motions.[1–3]

As is the case with any marine system, this system is expected to operate in a highly variable environment. Steady-state configurations and dynamic response will be affected by day-to-day fluctuations in wind, sea state, underwater currents, and the towing vessel throttle settings, for example. Some fluctuations (e.g., underwater currents) affect the motion of the system directly, whereas others (e.g., sea state) affect the system indirectly by influencing the ship's motion. Other disturbances to the system include unsteady vehicle hydrodynamics and sensor noise. An automatic control system that has good disturbance rejection and noise suppression characteristics is required to minimize position and attitude fluctuations in such an environment. Moreover, operation of the system at various speeds, scopes, and depths means that controllers may be designed for many different operating conditions (set points). For continuous operation of the system between set points, gain scheduling may be required.

One approach to satisfying the preceding performance requirements is to design two independent controllers for each set point—one for depth and one for roll. The need for depth control is obvious; roll control is an indirect attempt at keeping the vehicle as deep and at as constant a position (relative to the towing vessel) as possible. These controllers may be single input/single output (SISO) or multi-input/single output systems. The output of the depth controller will be the elevator angle; the input may be depth, depth and depth rate, depth and pitch angle, etc. The output of the roll controller will be the aileron angle; the input may be only the roll angle, or it may be the roll angle and roll rate.

For systems that have side-looking sonar, however, it may be necessary to control vehicle yaw rate as well. This may be done by designing a separate yaw rate controller or by designing a coupled roll/yaw rate controller. This latter approach results in a multi-input/multi-output (MIMO) controller that will provide aileron and rudder angle changes given measurements of roll angle and yaw rate. Reference 3 presents the development of a depth controller for a towed vehicle using linear quadratic regulator (LQR) theory.

Software tools play a very important role in the design and testing of towed systems and their controllers. Models of the towed system (plant) dynamics can be used in conjunction with programs like MATLAB™ (Ref. 4) to provide quick assessments of changes in towed system and controller designs on overall system performance. Nonlinear dynamic models can be used to test system performance in the vicinity of set points and as it is driven far from those points. A well-integrated set of software tools can greatly facilitate this process. Reference 5 presents a useful set of software tools to aid in the design of underwater tethered cable systems.

This paper focuses on the use of MATLAB and the nonlinear towed system dynamics program DYNTOCABS[6] (available to industry from the U.S. Navy) to design and test single and multivariable linear controllers for underwater towed systems. The controllers were designed using the linear quadratic Gaussian/loop transfer recovery (LQG/LTR) design methodology.[7]

Control Design Procedure

The procedure used in this paper for controller design is displayed in Fig. 1. This section provides some of the details behind each step of this process. The process makes use of two existing computer programs. DYNTOCABS is used to model the nonlinear dynamics of towed cable systems. It can be used to simulate the open- or closed-loop dynamics of the system and can also provide linearized models of the open-loop plant at various operating points. MATLAB is used to compute reduced-order linear models, generate frequency response plots, form composite plant models with

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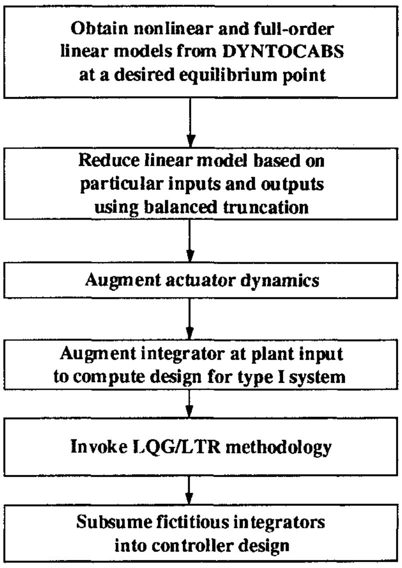

Fig. 1 Design procedure overview.

integrators for command following and actuator models, apply the LQG/LTR (or other) design methodology, and perform linear dynamic simulations. Application of the design procedure and testing of the resulting controllers is given in the next section.

Dynamic Model of Towed System

Many approaches are available that may be used to model towed system dynamics. In this study, the cable is modeled as a series of rigid links connected end-to-end with frictionless spherical joints; the mass and forces on these links are distributed at the connecting joints.[8,9] Towed vehicles are modeled as three-dimensional bodies with mass and inertia. The cable is connected to some arbitrary point on the vehicle with a spherical joint. The hydrodynamic forces on the towed vehicles are calculated from a set of hydrodynamic coefficients that include the effects of lift, drag, and added mass. Normal and tangential drag and side (lift) forces are included on the cable. The full nonlinear and linearized equations of motion of this type of model are generated by DYNTOCABS (see the Appendix). The linearized equations contain the cable/vehicle system dynamics but not actuator dynamics; these effects are added after the model reduction process is complete.

Model Reduction

The linearized equations of motion for the model just described are generally of high order, say roughly 100 state variables. Application of the LQG/LTR control theory to this plant results in a controller of the same order. Analysis of such high-order systems is both numerically unreliable and computationally intensive. The order of the controller can be kept small by either reducing the order of the plant prior to application of the design methodology or by reducing the order of the high-order controller. In this work the former approach was applied.

There are a number of ways to produce low-order approximations of high-order linear systems; one technique is called balanced truncation.[10,11] In this technique a similarity transformation is generated for the full-order system that balances the controllability and observability of the system. In this process it also reorders the states of the system from the most controllable and observable to the least. Approximations to the original system are then found by ignoring states that are the least controllable and observable. Any number of states can be ignored. Note that balanced truncation will yield different results depending on the specific inputs and outputs chosen for the full-order system. This is an important design issue, because the linearized equations of motion show a natural decoupling between in-plane (depth and pitch) and out-of-plane (roll and yaw) dynamics. So, for depth control, it would not be appropriate to obtain a reduced-order model based on inputs that affect out-of-plane dynamics, such as aileron or rudder. Finally, Bode diagrams and the step and impulse responses of the original and low-order systems can be compared to check the degree of fidelity of the low-order model.

Actuator Dynamics and Command Following

Prior to invoking the LQG/LTR design methodology, the reduced-order linear model is augmented with an integrator and an actuator model. The integrator allows the resulting controller to track commands away from steady state.[12,13] This is an important part of the controller's capabilities, given the variability of the environment. In particular, for a given tow speed and scope, the actual depth or attitude of the vehicle will be slightly different from day to day, making it impossible to know a priori the actual equilibrium configuration. The model of actuator dynamics is equally important. Under no load conditions vehicle flaps may respond rapidly to controller commands; however, under tow, flap response can be much slower, especially at high speeds. Slowing of the flap response may not be a problem, because along with the slowing comes an increase of control authority at higher speeds. However, because finite actuator bandwidths can result in significant effective time delays, good controller designs must take actuator dynamics into account. The actuator dynamics introduced under loading conditions can be nonlinear and difficult to model. In this work, the actuators were simply modeled as first-order linear systems, with slow time constants.

Design Methodology: LQG/LTR

It is well known that LQRs and Kalman filters individually exhibit excellent robustness properties.[12] Unfortunately, the LQG optimal controller, which combines LQR state feedback and the Kalman filter, fails to guarantee any such properties. The LQG/LTR method described briefly in this section allows the designer to set the properties of the Kalman filter to match the specifications of the towed system of interest and to recover these properties within a new model-based controller.[7] As presented, the method described next is more specifically referred to as "LTR at the plant output."

For a better understanding of the procedure, it is helpful to make note of the following. First, consider the block diagram of the closed-loop system as shown in Fig. 2. By cutting the block at the plant output, it can be shown that the open-loop transfer function is

$$L(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}\boldsymbol{K}_c(s\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}_c + \boldsymbol{K}_f\boldsymbol{C})^{-1}\boldsymbol{K}_f \quad (1)$$

where $\boldsymbol{K}_c$ and $\boldsymbol{K}_f$ are the LQR state-feedback and Kalman filter gains, respectively, and $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are the plant state-space matrices. If the plant has no nonminimum phase zeros and if the state quadratic cost matrix $\boldsymbol{Q} = \boldsymbol{C}^T\boldsymbol{C}$ and the control quadratic cost matrix $\boldsymbol{R} = \rho\boldsymbol{I}$, then it can be shown that

$$\lim_{\rho \to 0} L_\rho(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{K}_f \quad (2)$$

where L_ρ is the LQG loop transfer function parameterized by ρ. The right-hand side of this equation is the loop transfer function of the Kalman filter, functioning as an independent unit. Therefore, in this limiting process, the desired properties of the Kalman filter can be recovered. Note that to recover the desired properties at all frequencies, the system must have the same number of inputs as outputs, and it must be minimum phase. Care must be taken not to introduce nonminimum phase zeros in the model reduction process;

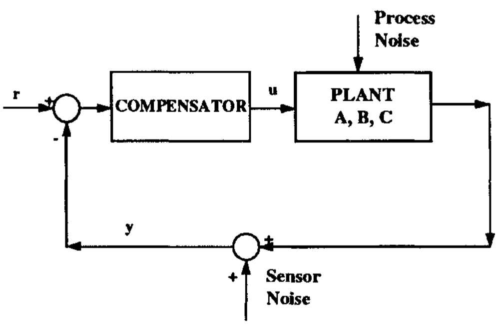

Fig. 2 Closed-loop plant.

otherwise the effectiveness of the LQG/LTR procedure is artificially limited.[7] In this case, the frequency response of the Kalman filter loop transfer function can be recovered only up to approximately one decade below the frequency associated with a nonminimum-phase zero.

We now present a method for choosing the design parameters for the desired Kalman filter that effectively allows the user to prescribe the singular values of the loop transfer function of the Kalman filter [right-hand side of Eq. (2)]. The method assumes that integrators are to be augmented into the design plant for tracking commands away from steady state. The state-space representation for the system with integrators at the plant input is

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 0 \\ A & B \end{bmatrix} \bar{x} + \begin{bmatrix} I \\ 0 \end{bmatrix} u \tag{3}$$

$$\bar{y} = [0 \quad C]\bar{x} \tag{4}$$

If A is invertible, and if the process noise covariance matrix W is set to I, the measurement noise covariance matrix V is set to μI, and the process noise input matrix Γ is chosen to be

$$\Gamma = \begin{bmatrix} -(CA^{-1}B)^{-1} \\ A^{-1}B(CA^{-1}B)^{-1} \end{bmatrix} \tag{5}$$

then all of the singular values of the sensitivity transfer function (corresponding to the loop transfer function of the Kalman filter) are equal at a given frequency ω and can be calculated using the equation

$$\sigma(\omega) = \left(\frac{\mu\omega^2}{\mu\omega^2 + 1}\right)^{\frac{1}{2}} \tag{6}$$

When plotted on a log-log scale, this function exhibits a +20 dB/decade slope up to $\omega_c = \mu^{-1/2}$ rad/s, after which the function rolls off and approaches 0 dB. By choosing the value of μ wisely, the designer can ensure that the Kalman filter has good noise and disturbance rejection properties.

Note that we have the freedom to choose W, V, and Γ as before because we are using the resulting Kalman filter as a target dynamical system to be recovered through LTR. This is in contrast to traditional Kalman filter design where W, V, and Γ are parameters that describe how noise enters the system and may not be selected arbitrarily. Also note that the assumption that A is invertible is purely for mathematical convenience; however, this happens to be true in the linearized models we used for towed systems.

Design Summary

The LQG/LTR procedure outlined earlier is a four-step process as summarized next.

1) Augment integrators into the input of the original reduced-order plant.

2) Choose μ, W, V, and Γ so that the target loop $L_{\text{target}} = C(sI - A)^{-1}K_f$ represents a desirable loop transfer function.

3) Recover the target loop transfer function by letting $Q = C^T C$ and $R = \rho I$ and computing K_c while letting $\rho \to 0$.

4) Augment integrators into the output of the compensator so that it can be implemented with the original plant (full-order linear, nonlinear, or real system).

Model-Based Testing of Controller Designs

The procedures outlined in the preceding sections can be used to generate controllers for marine towed vehicles. Before implementing a controller design and testing it at sea, it should be tested using the full-order linear and nonlinear models. Using these models the robustness of the controller can be tested against model uncertainty, sensor noise, tow vessel motion disturbances, unsteady vehicle hydrodynamics, flap angle and rate limits, actuator backlash, nonlinear vehicle hydrodynamics, etc. Models of towed systems have many sources of uncertainty including actuator dynamics, cable drag functions, and vehicle hydrodynamic coefficients. New models can be generated using reasonable variations in the coefficients of the original model. Sensor noise can be modeled with band-limited Gaussian noise. Disturbances to vehicle depth and attitude are modeled with low-frequency noise; disturbances to their rates generally require higher-frequency noise. Tow vessel motion disturbances are low-frequency disturbances and may be treated either with random (Gaussian) or harmonic functions. Unsteady vehicle hydrodynamics are difficult to model accurately; one simple method of introducing these effects is through low-frequency, band-limited Gaussian flap angle disturbances. These disturbances are added directly to controller flap commands. The sensitivity of the controller to these types of realistic effects must be evaluated.

Application to Towed Vehicle Control

This section shows results generated for an existing underwater vehicle towed at 12 kn (20.26 ft/s) from a 567-ft towing cable. The vehicle has a wing for generating depressive force and a tail with flaps for control. The flaps are assumed to provide three independent logical functions: aileron, rudder, and elevator. The cable has drag-reducing fairing covering the last 200 ft of cable adjacent to the vehicle. In the design process, it is assumed that the actuators driving the flaps are quite sluggish; they are modeled as first-order systems with a time constant of roughly 1 s. The controllers are implemented in discretized form assuming a sampling frequency of 10 Hz. All noise is low-pass filtered with a cutoff at 5 Hz to avoid aliasing.

Results are shown for two different LQG/LTR controller designs, referred to in the following as cases 1 and 2. The case 1 design is viewed by the authors as providing a reliable means of towed vehicle control; however, the case 2 design is considered experimental and is still under development. These two cases are shown to highlight the use of software such as DYNTOCABS for nonlinear model-based testing of controller designs.

The controllers for both cases 1 and 2 consist of two separate (uncoupled) LQG/LTR designs, one for controlling motion in the towing plane (cable/vehicle plane) and one for controlling motion out of the towing plane. The in-plane controller is the same for both cases; it generates elevator commands in response to measured depth variations. The out-of-plane controllers, however, are different for each case. The case 1 out-of-plane controller generates aileron commands in response to roll angle deviations, whereas the case 2 design generates aileron and rudder commands in response to deviations in roll angle and yaw rate deviations. Maintaining zero roll helps to keep the vehicle more directly behind the towing vessel, and maintaining a zero yaw rate minimizes yaw motions. Because of the presence of currents, it is not possible to know a priori what the at-sea steady-state yaw and pitch angles of the vehicle should be, and so direct control of these variables is not considered here.

The cases 1 and 2 depth controllers and the case 1 roll controller are all based on plant models reduced (without actuator dynamics) to third order, the depth controllers using an in-plane model and the roll controller an out-of-plane model. After including a first-order actuator model and two integrators for each, the controllers are all sixth order. The case 2 roll/yaw rate controller is based on a plant model reduced to 10th order. After including actuators for rudder and aileron (2 total) and integrators (4 total), the controller is 16th order. It should be noted that the out-of-plane plant model used for the case 2 design is much larger than that for the case 1 design. This is because reduction to lower orders using balanced truncation introduces nonminimum-phase zeros that are detrimental to the LQG/LTR designs. For the depth controllers, the value of the design parameter μ is 20; for the roll and roll/yaw rate controllers, μ is 1. In all cases, the value of the parameter ρ is 0.001.

The controllers for both cases were thoroughly tested using the full-order linear model of the towed system and were found to perform well. They were then tested in a more realistic environment using the nonlinear model in DYNTOCABS. The results of some of these tests are summarized next.

The first set of results is for the case 1 design. The open-loop system is perturbed for 100 s with tow vessel motion disturbances, sensor noise, and control surface disturbances. The tow vessel motion disturbance is harmonic surge motion with an amplitude of 15 ft and a frequency of approximately 0.02 Hz. Noise and disturbances

Table 1 Noise and disturbance properties

Noise/disturbance	σ	f, Hz
Roll sensor, deg	1.0	5.0
Depth sensor, ft	1.0	5.0
Aileron angle	0.5	0.01
Rudder angle, deg	0.5	0.01
Elevator angle	1.0	1.0

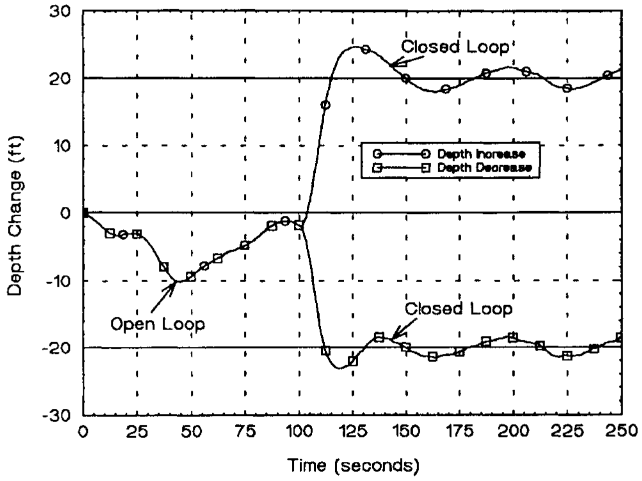

Fig. 3 Case 1 depth responses.

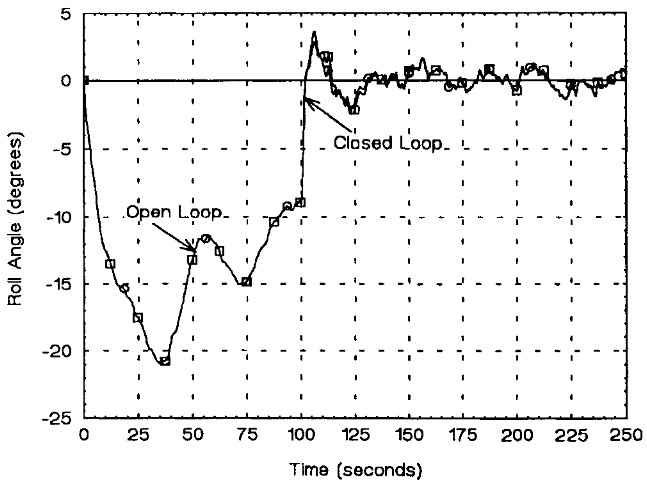

Fig. 4 Case 1 roll responses.

are modeled as Gaussian processes, band-limited from zero to some cutoff frequency. The standard deviations and cutoff frequencies used are shown in Table 1. The flap actuators are modeled as first-order linear systems with time constants of 1 s.

At 100 s, the controllers are activated; commands are given to change steady-state depth by 20 ft and to reduce the roll angle to zero. Figures 3 and 4 show the depth and roll plots for both 20-ft depth increases and decreases. Even though the system is far from steady state when the controllers are activated, they seem to have no trouble responding to the depth and roll commands even in the presence of noise and disturbances. Moreover, the closed-loop system fluctuations are well below those of the open-loop system.

The case 2 controller was tested under the same conditions as shown earlier for case 1. The system was perturbed for 100 s and then the controller was activated. At this time three commands were executed simultaneously: change depth, return to zero roll, and return to zero yaw rate. Whereas the in-plane command is as it was in case 1, the out-of-plane command is very different. In case 1 the yaw angle is allowed to freely change as the roll angle is commanded to zero; in case 2 the yaw angle is held constant (zero yaw rate).

The baseline test of the case 2 controller used the same noise, disturbances, and actuator dynamics as in the case 1 test; in addition, the rudder actuator was assumed to respond instantaneously and the yaw rate noise was assumed to be zero. Under these conditions, the case 2 results for depth and roll response are very similar to those shown in Figs. 3 and 4. However, as expected, the yaw response is different. Figure 5 compares the yaw responses of the two cases. The case 2 controller allows the system to avoid large deviations in yaw angle such as occurred for case 1 just after the controller was activated at 100 s. Unfortunately, it introduced an undesirable 0.25-Hz oscillation as well.

This design was further tested by introducing rudder actuator dynamics and yaw rate sensor noise. Figures 6 and 7 compare the

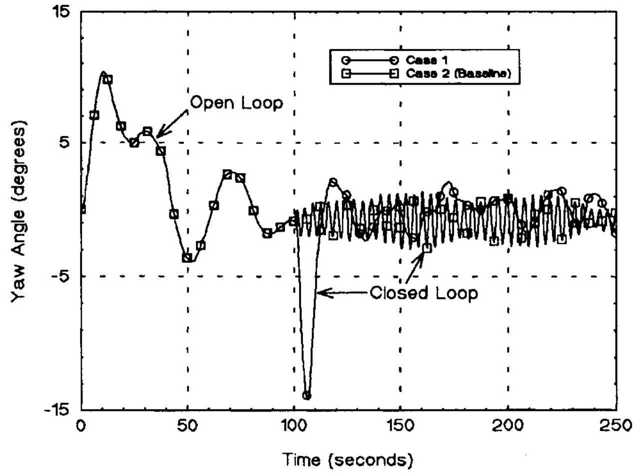

Fig. 5 Comparison of yaw responses: cases 1 and 2 baseline responses.

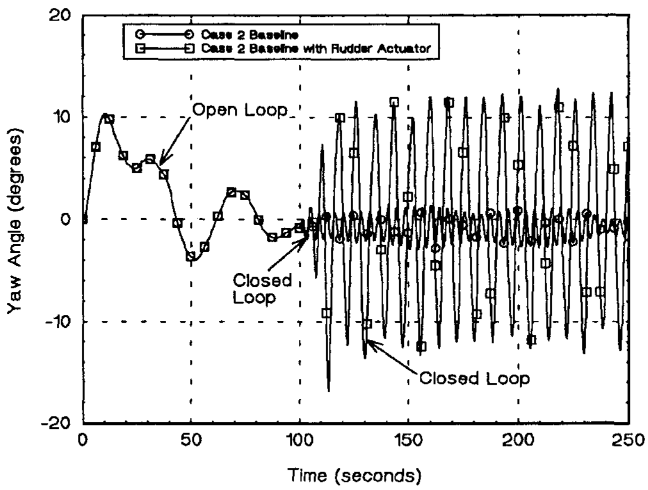

Fig. 6 Case 2 yaw response: baseline and rudder actuator tests.

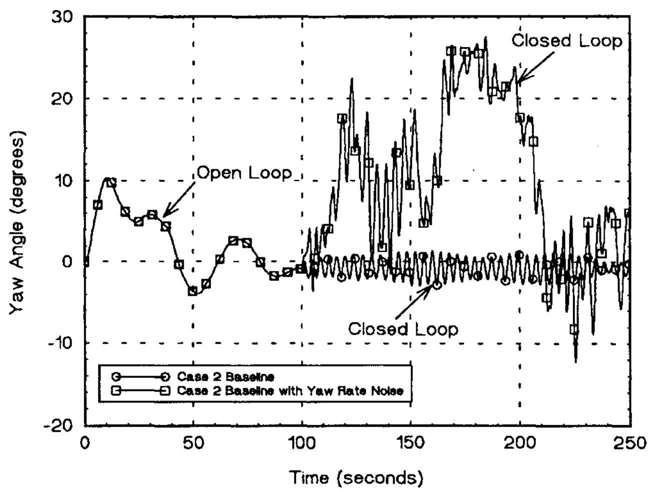

Fig. 7 Case 2 yaw response: baseline and sensor noise tests.

response of the baseline system with each of these systems. Clearly, in both cases, the response of the system is now unacceptable. Poor performance is also noted in the roll and depth responses of the system. Modifications to the case 2 controller design that will make it less sensitive to actuator dynamics and sensor noise are currently under consideration.

Conclusions

For towed marine sonar systems to be effective in the highly variable environment at sea, closed-loop control systems are required. For towed systems, the linear models that are used for the design of these controllers change with tow speed, cable scope, and vehicle depth. Each of these set points may require the development of a separate controller. This paper presents a procedure for the development and model-based testing of controllers for these systems that combines the use of existing software and the well-known LQG/LTR control methodology. Programs like MATLAB are used to generate reduced-order linear plant models and frequency response plots, to form composite plant models with integrators and actuator models, to apply the LQG/LTR design methodology, and to test the controller designs with the full-order linear models. Programs like DYNTOCABS are used to generate nonlinear dynamic models and linearized plant models for MATLAB and to test the robustness of controller designs using the nonlinear models. Programs such as these help to automate the process of controller design and testing; this shortens the time required to generate designs and allows the designs to be more thoroughly tested before sea trials.

Results are shown for two different cases of towed vehicle control. In each case the controllers were tested thoroughly using the full-order linear model and were found to perform well. Then they were tested using the nonlinear model in DYNTOCABS. In the first case, the depth and roll angle are controlled with two independent SISO controllers. Depth is controlled using an elevator and roll with an aileron. The results show this design to be somewhat robust, and the authors believe this is a reliable means of control. Sea trial results of existing vehicles at least partially validate this position. The second case is considered by the authors to be more experimental in nature. In this case, the depth is controlled by a SISO controller (same as case 1 design), and the roll angle and yaw rate are controlled by an independent MIMO (2×2) controller. Results for this case show that the controller is not robust and therefore not suitable for use in its present form. This design is still under development to make it less sensitive to actuator dynamics and sensor noise.

The methodology and software discussed herein are expected to provide the control system designer with the tools necessary to efficiently design and test various approaches for towed vehicle control. The designs presented are intended to illustrate the use of these tools and are in no way considered by the authors to be the best designs. It may be possible to generate better designs by changing any number of the many parameters at the designer's command. For example, larger control surfaces with faster actuators may help the case 2 design; recall, that in the design phase, the actuators were assumed to be very sluggish (time constants of about 1 s). Finally, however, note that the controller designs are based on linear models and their performance will be limited by the range of accuracy of those models.

Appendix: Modeling Towed System Dynamics

Lumped Parameter Model

A towed cable system is assumed to be a single or multiple branched cable system with towed vehicles. It is assumed to be pulled by a ship or helicopter from a single tow point. The motion of the system tow point is arbitrary. The cable and its branches form an open tree system with no closed kinematic chains. Each length of cable may have different physical and hydrodynamic properties. The towed vehicles are three dimensional and may be actively controlled, or they may be passive.

The cable segments are assumed to be axially inextensible and laterally flexible; they are modeled by a series of rigid links connected by frictionless spherical joints. The masses and fluid and gravitational loads are assumed to be uniformly distributed over each link. Each is then halved and lumped at the connecting joints, making the links two-force members. The towed vehicles are assumed to be rigid bodies with mass and inertia. Each is connected to a single cable link by a frictionless spherical joint; this connection point is arbitrarily located on the body. Normal and tangential fluid drag, side (lift), weight, and buoyancy forces are included for each cable link. The towed vehicles are assumed to have separate mass and buoyancy centers. Their drag, lift, and added mass forces are calculated from a user-supplied set of hydrodynamic coefficients. The motion of a towed vehicle's control surfaces may be specified using open- or closed-loop procedures.

System Configuration

The position of the system at any time is described relative to a reference frame that represents the mean motion of the towing vessel, called the mean ship frame. As time progresses, the mean ship frame is assumed to move in a horizontal plane relative to the inertial frame. Its motion is produced by specifying the ship's forward speed and heading angle as a function of time. This frame provides a convenient reference frame for describing the position of the system, especially during steady-state motions. The effects of wave-induced ship motion are included by specifying time-varying motion of the system tow point relative to the mean ship frame.

The position of the rest of the system is described by a vector θ of orientation angles measured relative to the mean ship frame. The cable links are described by two angles and the towed vehicles by three. Hence, there are $2NC + 3NT$ degrees of freedom in a model with NC cable links and NT towed vehicles.

Nonlinear Equations of Motion

Given some user-specified functions for the position of the mean ship frame and the system tow point and given an initial system configuration (θ and $\dot{\theta}$), the Cartesian coordinates $\boldsymbol{x}$ and Cartesian velocities $\dot{\boldsymbol{x}}$ of the mass centers of the towed vehicles and of the lumped masses are calculated from the equations of rigid-body kinematics as

$$\boldsymbol{x} = \boldsymbol{f}(\theta) \qquad \text{and} \qquad \dot{\boldsymbol{x}} = \boldsymbol{g}(\theta, \dot{\theta}) \tag{A1}$$

where $\boldsymbol{f}$ and $\boldsymbol{g}$ are nonlinear functions of θ. These are used to calculate the external force vector $\boldsymbol{Q} = \boldsymbol{h}(\boldsymbol{x}, \dot{\boldsymbol{x}})$ that includes weight, buoyancy, and fluid drag and lift.

Given these results, equations derived from Newton's law are then combined with constraint equations (defining the rigidity of the cable links and towed vehicles) to produce a set of linear algebraic equations that are solved for the cable tensions $\boldsymbol{t}$

$$\boldsymbol{T}(\theta)\boldsymbol{t} = \boldsymbol{q}(\boldsymbol{Q}) \tag{A2}$$

where the elements of the coefficient matrix $\boldsymbol{T}(\theta)$ are nonlinear functions of θ. Note that these equations are tridiagonal for cable segments with no branches or towed vehicles. Branch points and towed vehicles introduce only a few values outside the tridiagonal band, and so these equations can be solved very efficiently by first eliminating the values outside the band and then using a standard tridiagonal solution method.

These internal cable tensions are then returned to the equations derived from Newton's law to calculate the Cartesian accelerations $\ddot{\boldsymbol{x}} = \boldsymbol{r}(\boldsymbol{Q}, \boldsymbol{t})$. These values are then used in the equations of rigid-body kinematics to calculate the second derivatives of the orientation angles $\ddot{\theta} = \boldsymbol{s}(\theta, \dot{\theta}, \ddot{\boldsymbol{x}})$. Finally, given θ, $\dot{\theta}$, and $\ddot{\theta}$, the solution can be advanced in time to $t + \Delta t$ using some numerical integration method. DYNTOCABS contains either Runge–Kutta or predictor-corrector schemes for this purpose.

Linearized Equations of Motion

During steady forward or steady turning motion and in the absence of external disturbances, the system can exhibit steady-state equilibrium positions. In these situations, the orientation angles relative to the mean ship frame remain constant. DYNTOCABS can determine these positions and then linearize the preceding nonlinear equations about these positions. The linearized equations are provided in state-space form and may be used in the design of towed vehicle controllers.

DYNTOCABS uses a free-body-diagram approach to successively find the equilibrium angles, starting with the bodies at the ends of the cable branches and moving toward the system tow point. Care must be taken to complete all branches before passing through a branch point. Using this approach the equilibrium angles are determined efficiently by solving a series of small sets (two equations for a cable link and three for a towed vehicle) of nonlinear algebraic equations. This procedure is complicated somewhat for partially submerged systems or systems undergoing circular motion. In these cases this procedure must be repeated until convergence to the final solution occurs.

To describe the motions of the system that result from small disturbances around the steady-state positions, the nonlinear equations described earlier may be linearized about those positions. Expanding the nonlinear equations in a Taylor series and eliminating nonlinear terms give linear equations in state-space form,

$$\dot{y} = Ay + Bu \tag{A3}$$

In this equation,

$$y = \begin{Bmatrix} \Delta\theta \\ \Delta\dot{\theta} \end{Bmatrix}$$

and $\boldsymbol{u}$ contains the external inputs such as deviations in tow point motion (uncontrollable) and in towed vehicle control surface settings (controllable). These equations may be supplemented with a set of linearized kinematic equations $z = Cy$ where z is a vector of desired output variables. DYNTOCABS numerically perturbs the nonlinear equations (described earlier) and uses second-order central differences to estimate the entries in the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$. Note that since the steady-state motions are stationary, these matrices are all constant.

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